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GERBES, 2-GERBES AND SYMPLECTIC FIBRATIONS

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ABSTRACT. Let $p : P \to N$ be a symplectic bundle whose typical fiber is the symplectic manifold (F, ω). McDuff has defined a subgroup $\mathrm{Ham}^s(F, \omega)$ of the group of symplectic automorphisms of (F, ω) and has shown that the cohomology class $[\omega]$ extends to P if and only if p has a $\mathrm{Ham}^s(F, \omega)$ reduction. The purpose of this paper is to interpret the result of McDuff using gerbe theory. We define fundamental gerbes in symplectic geometry which allows us to define a 2-gerbe which represents the geometric obstruction to lift ω to P. Using these gerbes, we define a geometric quantization of symplectic manifolds.

1. Introduction. A *symplectic fibration* $P \to N$ is a differentiable fibration whose typical fiber is the closed connected symplectic manifold (F, ω), and such that there exists a trivialization (U_i, g_{ij}), such that $g_{ij}(u)$ is a symplectic automorphism of the fiber over u, endowed with a symplectic structure ω_u, symplectomorphic to (F, ω). We suppose that the cohomology class $[\omega_u]$ of ω_u is fixed. The theory of symplectic bundles has been studied by different authors, see [**8, 9, 12, 16**]. One purpose of the paper [**16**] is to determine whether the structural group of the symplectic bundle can be reduced to the Hamiltonian group of (F, ω), that is, whether there exists a symplectic bundle $P' \to N$ isomorphic to P, whose coordinate changes $g'_{ij}(u)$ are Hamiltonian automorphisms of the fiber above u; such a reduction will be called a *Hamiltonian structure*, or a Ham-reduction. In [**16**], it is shown that the existence of such Hamiltonian reductions on a finite cover of N is equivalent to the following two conditions:

(i) There exists a closed 2-form Ω defined on P whose cohomology class $[\Omega]$ extends $[\omega]$. This means that the restriction to the fiber above u of the cohomology class $[\Omega]$ is the cohomology class $[\omega]$. Following McDuff, we will call the form Ω a *closed connection form.*

(ii) Let $\mathrm{Symp}\,(F, \omega)_0$ be the connected component of the group of symplectomorphisms $\mathrm{Symp}\,(F, \omega)$, of (F, ω). The symplectic bundle is

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