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SOME APPLICATIONS OF GASPER'S BIBASIC SUMMATION FORMULA

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ABSTRACT. A new bibasic summation formula for hypergeometric series is found by a special inversion formula and it is applied to derive a class of transformation formulas and summation formulas for basic hypergeometric series only by elementary methods. The ordinary hypergeometric limits of these formulas are also obtained.

1. Introduction. All the notation and terminology is adopted from [**9**]. The (generalized) hypergeometric series is defined by

$$ {}_{r+1}F_r \left[\begin{matrix} a_1, \dots, a_r \\ b_1, \dots, b_s \end{matrix} ; z \right] = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_r)_n}{n!(b_1)_n \cdots (b_s)_n} z^n, $$

where the rising factorial $(a)_n$ is given by

$$ (a)_n := a(a+1)(a+2)\cdots(a+n-1), \quad n \geq 1, \quad (a)_0 := 1. $$

The gamma function can be used to extend to rising factorials by defining $(a)_\beta = \lim_{\gamma\to a} \Gamma(\gamma+\beta)/\Gamma(\gamma)$, β arbitrary. A hypergeometric series ${}_{r+1}F_r$ is called very well-poised if $a_i + b_i = 1 + a_0$ for $i = 1, 2, \dots, r$, and among the parameters a_i occurs $1 + a_0/2$. We use the standard abbreviation for very well-poised hypergeometric series, ${}_{r+1}V_r(a_0; a_2, a_3, \dots, a_r; z)$

$$:=_{r+1} F_r \left[\begin{matrix} a_0, 1+a_0/2, a_2, a_3, \dots, a_r \\ a_0/2, 1+a_0-a_2, 1+a_0-a_3, \dots, 1+a_0-a_r \end{matrix} ; z \right] $$

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