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OSCILLATION THEOREMS RELATED TO AVERAGING TECHNIQUE FOR SECOND ORDER EMDEN-FOWLER TYPE NEUTRAL DIFFERENTIAL EQUATIONS

ZHITING XU

ABSTRACT. Some oscillation theorems are established by the averaging techniques for the second order Emden-Fowler type neutral delay differential equation

$$(r(t)x'(t))' + q_1(t)|y(t-\sigma_1)|^{\alpha-1}y(t-\sigma_1)$$

$$+q_2(t)|y(t-\sigma_2)|^{\beta-1}y(t-\sigma_2) = 0,\ t \geq t_0,$$

where $x(t) = y(t) + p(t)y(t-\tau)$, τ, σ_1 and σ_2 are nonnegative constants, $0 < \alpha < 1$, $\beta > 1$, and r, p, q_1, $q_2 \in C([t_0,\infty),\mathbf{R})$. These theorems obtained here extend and improve some known results. In particular, two interesting examples that point out the applications of our results are also included.

1. Introduction. In this paper, we study the problem of oscillation of the second order Emden-Fowler type neutral delay differential equation

$$(1.1)\quad (r(t)x'(t))' + q_1(t)|y(t-\sigma_1)|^{\alpha-1}y(t-\sigma_1) + q_2(t)|y(t-\sigma_2)|^{\beta-1}y(t-\sigma_2) = 0, \quad t \geq t_0,$$

where $x(t) = y(t) + p(t)y(t-\tau)$, and the following conditions are assumed to hold:

(A1) τ, σ_1 and σ_2 are nonnegative constants, $0 < \alpha < 1$, $\beta > 1$;

(A2) r, q_1, $q_2 \in C([t_0,\infty),\mathbf{R}^+)$, and $\int^\infty 1/r(s)\,ds = \infty$, $\mathbf{R}^+ = (0,\infty)$;

(A3) $p \in C([t_0,\infty),\mathbf{R})$, and $-1 < p_0 \leq p(t) \leq 1$, p_0 constant.

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