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ON A RIEMANNIAN INVARIANT OF CHEN TYPE

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ABSTRACT. In [**6**] we proved Chen's inequality regarded as a problem of constrained maximum. In this paper we introduce a Riemannian invariant obtained from Chen's invariant, replacing the sectional curvature by the Ricci curvature of k-order. This invariant can be estimated, in the case of submanifolds M in space forms $\widetilde{M}(c)$, varying with c and the mean curvature of M in $\widetilde{M}(c)$.

1. Introduction. We consider a Riemannian manifold (M, g) of dimension n, and we fix the point $x \in M$. The scalar curvature is defined by

$$\tau = \sum_{1 \le i < j \le n} R(e_i, e_j, e_i, e_j),$$

where R is the Riemann curvature tensor of (M, g) and $\{e_1, e_2, \dots, e_n\}$ is an orthonormal frame in $T_x M$.

Let L be a vector subspace of dimension $k \in [2, n]$ in $T_x M$. If $X \in L$ is a unit vector, and $\{e'_1, e'_2, \dots, e'_k\}$ is an orthonormal frame in L, with $e'_1 = X$, we shall denote

$$\mathrm{Ric}_L(X) = \sum_{j=2}^{k} k(e'_1 \wedge e'_j),$$

where $k(e'_1 \wedge e'_j)$ is the sectional curvature given by $\mathrm{Sp}\,\{e'_1, e'_j\}$.

Using the Ricci curvature of k-order at the point $x \in M$,

$$\theta_k(x) = \frac{1}{k-1} \min_{\substack{L,\ \dim L = k \\ X \in L,\ \|X\| = 1}} \mathrm{Ric}_L(X),$$

we define the invariant

$$\delta_k(M) : M \to R,$$
$$\delta_k(M) = \tau - \theta_k.$$

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