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COMPOSITION OF OPERATORS IN ORLICZ SPACES

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ABSTRACT. In this work we find sharp conditions for boundedness on Orlicz spaces of the composition of j operators, each one being of restricted weak type (p,p) for some $p>1$, and of strong type (∞,∞). Particularly, we find necessary and sufficient conditions to obtain modular inequalities for the j-times composition of the Cesàro maximal function of order α. With this approach we treat a kind of strong maximal function related to Cesàro averages over n-dimensional rectangles.

1. Introduction. Let (Ω,μ) be a measure space and $\mathfrak{M}(\Omega)$ the space of measurable functions. Let $j\in\mathbf{N}$ and $T_1,T_2,\dots,T_j$, be sublinear operators defined on $\mathfrak{M}(\Omega)$, so that all of them are of strong type (∞,∞) and, for a given $p\geq 1$, of restricted weak type (p,p), $1\leq k\leq j$, that is, there exist constants A_k and B_k such that for any measurable function $f\in\mathfrak{M}(\Omega)$

$$\|T_k f\|_\infty \leq A_k \|f\|_\infty,$$

and

$$\mu_{T_k f}(s) \leq \left(\frac{B_k}{s}\int_0^\infty \mu_f^{1/p}\right)^p \quad \text{for all} \quad s>0,$$

where μ_g denotes the distribution of a measurable function g.

It is well known that for a sublinear operator T these two conditions can be expressed in just one inequality, namely,

$$\mu_{Tf}(t) \leq \left(\frac{C}{t}\int_{t/C}^\infty \mu_f(s)^{1/p}\,ds\right)^p \quad \text{for all} \quad t>0, \tag{1}$$

where C is a constant independent of f and t.

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