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ON THE UPWARD EMBEDDING ON THE TORUS

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ABSTRACT. An upward embedding of a digraph on an embedded surface is an embedding of its underlying graph on that surface such that all arcs are represented by monotonic curves that point to a fixed direction. In this paper we study the concept of upward embedding on the torus. We shall introduce a partition of the arcs of a digraph and based on that we shall investigate some characteristics of digraphs that admit upward embedding on the horizontal torus. We also present a polynomial time algorithm for upward embedding testing of single source and single sink digraphs on the horizontal torus. We shall investigate the relation between the vertical and the horizontal tori with respect to the upward embedding.

1. Introduction. Graph embeddings and their generalization on surfaces have many applications, such as VLSI layout and graphical representations of a poset. In fact, it is customary and convenient to draw a diagram of an ordered set on the plane, whether or not edges cross. We may also wish to draw them on other surfaces, especially if this avoids the crossing of edges. In this paper we deal with the upward embedding of digraphs, which is defined as follows.

An upward embedding of a digraph D on an embedded surface S is an embedding of its underlying graph on the surface such that all arcs are represented by monotonic curves that point to a fixed direction.

The study of upward embedding on surfaces has been motivated by graph embedding, and topological graph theory, whose literature is extensive (cf., for example, [**8, 15**]). However, there are major differences between graph embedding and upward embedding of digraphs. For instance, all genus one orientable surfaces are topologically homeomorphic to a ring torus, which in turn, from the point of view of graph embedding is equivalent to horizontal and vertical tori. But in this paper we show that in upward embedding the critical points of these

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