ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 19, Number 4, Fall 1989

CONTINUED FRACTION TAILS AND IRRATIONALITY

R.M. HOVSTAD

ABSTRACT. A theorem on irrational numbers using tails is proved. Two examples are given. A comment is made on a classical result in the light of the method used.

1. Introduction. Tails of continued fractions have recently been in focus in continued fraction theory, see [**1**].

Consider the continued fraction

$$\mathbf{K}_{k=1}^{\infty} \frac{a_k}{b_k} \tag{1}$$

where $a_k \neq 0$ and b_k are complex numbers for $k \geq 1$. Then we call the continued fractions

$$T_k = \mathbf{K}_{i=k}^{\infty} \frac{a_i}{b_i} \tag{2}$$

for $K \geq 1$ the tails of (1). In the present treatment we consider only the case where the elements a_k and b_k in (1) are integers. A closer examination of the proof of a well known classical theorem on irrationality shows that the concept of tail can be used with advantage and provides a more transparent access to this irrationality result, see [**2**, p. 56].

A comment at the end of the present treatment will make this explicit and show that this irrationality result is almost trivial in the light of tails. We will illustrate the tail approach by giving a theorem on irrationality below. This theorem contains two possible outcomes precisely like the classical result in [**2**]. This means that (1) is irrational in certain situations and rational in the opposite situations. The theorem gives complete information.

As an illustration, one example of each situation is given. We now formulate the core of the tail method. Choose a number $x_0 \neq 0$. Define

$$x_k = T_k x_{k-1} \tag{3}$$

Received by the editors on December 30, 1986.