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GEOMETRIC MODULI FOR KLEIN SURFACES

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1. Introduction and statement of results. The analytic counterpart of a real algebraic curve of genus g is a Klein surface of genus g. That is a compact topological surface Σ^*, which is either non-orientable or has boundary components (or both), together with an analytic structure. This assertion is due to Norman Alling and Newcomb Greenleaf (cf [**1**]). The surface Σ^* is a topological model for the corresponding real curve. Observe that surfaces arising as topological models of real algebraic curves are never classical compact and oriented surfaces without boundary. There are also some other restrictions for the topological type of real curves of genus g. These constraints are not very complicated and we can easily compute the number of different topological types of real algebraic curves of genus g - that number is $(3g+4)/2$ (cf., e.g., [**3**, §2]).

The moduli problem is to give, for an algebraic curve, parameters or moduli which determine its isomorphism class. In the complex case the isomorphism classes of complex algebraic curves of a given genus g form an algebraic variety. The case of real algebraic curves is more complicated. The genus does not yet classify them even topologically. Hence, instead of considering all real algebraic curves of a given genus, we should consider all real algebraic curves of a given topological type Σ^*.

To exclude certain special cases we assume now that all real algebraic curves or Klein surfaces are of genus $g > 1$. That excludes the following Klein surfaces: the disk, the real projective plane, the annulus, the Möbius band and the Klein bottle.

It is now a formal simplification to consider the interior Σ of the surface (Σ^* instead of Σ^* itself. By a real algebraic curve of the topological type Σ we then mean a real algebraic curve of the type Σ^* in the sense of Alling and Greenleaf. These real algebraic curves are

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