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ON THE STRUCTURE OF EVEN UNIMODULAR EXTREMAL LATTICES OF RANK 40

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In memory of the late Professor Hel Braun

1. Introduction. Let $\Gamma_{8k}(k \geq 1)$ be the genus consisting of all equivalence classes of positive definite even unimodular quadratic lattices of rank $8k$. In an element L of Γ_{8k}, a vector x in L is called a $2m$-vector if x satisfies $(x,x) = 2m$, where $(\,,\,)$ is the inner product of L and $2m$ is an even integer. In obtaining a complete picture of the configurations of all equivalence classes in $\Gamma_{8k}(k \geq 4)$, the classes of lattices without 2-vectors would be a main obstacle.

In this paper, we study the subfamily $\Gamma_{40,0}$ of Γ_{40} consisting of all equivalence classes of lattices without 2-vector. As in [**7**], we use $\mathcal{L}_{2m}(L)$ (respectively, $\mathcal{L}_{2m_1+2m_2}(L)$) to denote the sublattice of L generated by all $2m$-vectors (respectively $2m_1$-vectors and $2m_2$-vectors) in L. In §2, we prove

THEOREM 1. *Let L be a lattice in $\Gamma_{40,0}$. Then we have*

$$L = \mathcal{L}_{4+6}(L).$$

In §3,we shall introduce the notion of the c-sublattice of a lattice in $\Gamma_{40,0}$. We expect this notion would play a role in the study of the structures of lattices in $\Gamma_{40,0}$, and also in $\Gamma_{32,0}$. However our present study of the c sublattice is merely a beginning of exploration.

We collect some standard notations used throughout the paper: $\mathbf{Q}$ is the field of rational numbers, $\mathbf{Z}$ is the ring of rational integers, $\mathbf{M}(1,k)$ (respectively $\mathbf{S}(1,k)$) is the linear space of modular (respectively cusp) forms of degree 1 and weight k, $\mathbf{E}_k(\mathbf{z})$ is Eisenstein series of degree 1 and weight k, $\Delta_{12}(\mathbf{z})$ is the normalized cusp form of degree 1 and weight 12. Special notations are explained in the appropriate places if necessary.

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