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u = 4 AND QUADRATIC EXTENSIONS

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1. Introduction. Throughout this paper L will denote a nonformally real field of characteristic $\neq 2$. By $u(L)$ we mean the *u-invariant* of L, i.e., $u(L) = \max\{n \in \mathbf{N}$: there exists an n-dimensional anisotropic quadratic form over $L\}$ (see [**16**, Chapter X]).

The motivation for our work is the following conjecture, which is part of the folklore of quadratic forms theory.

CONJECTURE 1.1. *If $a \in L$, then $u(L(\sqrt{a})) = u(L)$.*

In §2 we will present a couple of examples related to this conjecture in the particular case when $u(L) = 4$. Our strategy is to translate the condition $u(L) = 4$ into the Galois theory of L and then use some well known results on the cohomology of pro - 2 - groups. For the basic concepts and notation we use, the reader may consult [**16**] and [**28**].

One of our main tools will be the following important result. We let $\mathrm{cd}_p(G)$ denote the cohomological p - dimensional of the pro - p - group G (see [**28**, p. I–17]).

THEOREM 1.2. (SERRE [**30**]). *Let G be a pro-p-group that does not contain an element of order p and let H be an open subgroup of G. Then $\mathrm{cd}_p(G) = \mathrm{cd}_p(H)$.*

We now let $L(2) :=$ quadratic closure of L and $G_L := \mathrm{Gal}(L(2)/L)$. Then Theorem 1.2 (with $p = 2$) applies to $G = G_L$ since nonformally real fields L are characterized by the fact that G_L does not contain nontrivial involutions [**7**, Chapter 2, Theorem 3].

To our knowledge the first explicit connection between $\mathrm{cd}_2(G_L)$ and $u(L)$ was found by Ware in [**34**], where it was shown that $u(L) = 2 \Leftrightarrow$

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