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CLASSES OF QUATERNION ALGEBRAS IN THE BRAUER GROUP

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Elman and Lam investigated fields L such that the classes of quaternion algebras over L form a subgroup in the Brauer group $\mathrm{Br}(L)$ of the field L [**4**]. They made the following list of examples: L is a finite field, a local field, a global field, a field of transcendence degree ≤ 2 over $\mathbf{C}$, a field of transcendence degree 1 over $\mathbf{R}$, $\mathbf{C}((t_1))((t_2))((t_3))$, where $K((t))$ means the field of formal power series in t over the field K.

Elman and Lam found in their paper [**4**] that if L is a nonformally real field and the classes of quaternion algebras form a subgroup in the Brauer group $\mathrm{Br}(L)$, then

$$u(L) \in \{1, 2, 4, 8\}.$$

Here u means the so-called u-invariant of the field. (See [**4**], or [**6**; Chapter 11, Theorem 4.10]

DEFINITION 1. A field K is called *linked* if and only if the classes of quaternion algebras form a subgroup of the Brauer group $\mathrm{Br}(K)$. (See also Definition 4.3 in [**2**].)

In [**3**] it is proved that a formally real Pythagorean field F is linked if and only if F is SAP.

Our goal is to characterize all linked fields $L = F(\sqrt{-1})$, where F is formally real Pythagorean with finite chain length. This will generalize the sixth example above. We shall use the possibility to attach, to each order space X of finite chain length, some graded ring $R(X)$, which will be described explicitly in Definition 4. The main motivation for the introduction of $R(X)$ is given by Theorem 5 below.

We use standard notation such as can be found in [**5, 6**, and **8**]. For the reader's convenience, we shall recall just a bit of it.

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