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ON THE SCHARLAU TRANSFER

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Let F be a field of characteristic $\neq 2$, F_s a separable closure of F and $G_F = \mathrm{Gal}\,(F_s/F)$. The ring $A(F)$ of *monomial representations* of G_F is defined as follows: it is the Grothendieck ring of the category of pairs (E, E'), where E and E' are étale F-algebras and E' is a free E-algebra of rank 2 [**1**, III 2.2]. Such pairs are classified by homomorphisms of G_F in a wreath product $\mathbf{S}_n \int(\mathbf{Z}/2) \simeq O(n, \mathbf{Z})$ (*loc. cit.*), so $A(F)$ really depends only on G_F.

On the other hand, write (here) $W(F)$ for the Witt-Grothendieck ring of F (the Grothendieck ring of the category of non-degenerate quadratic forms over F). In [**1**, III. 2.6] a ring homomorphism $h : A(F) \to W(F)$ was defined; it may be described in (at least) two different ways:

(a) Let (E, E') be a generator of $A(F)$. Since $\mathrm{char}\, F \neq 2$, there is an $a \in E^*$ such that $E' = E[\sqrt{a}]$. Then $h(E, E')$ is the class in $W(F)$ of the quadratic form $q(x) = \mathrm{Tr}_{E/F} ax^2$.

(b) Let $\rho : G_F \to O(n, \mathbf{Z})$ be a homomorphism classifying (E, E'). Then $O(n, \mathbf{Z})$ maps naturally to a Galois-invariant subgroup of $O(n, F_s)$, hence to ρ is associated an element in the nonabelian cohomology set $H^1(G_F, O(n, F_s))$; via [**2**, p.162, Corollary 1], this element corresponds to a quadratic form $h(\rho)$.

Observe that $W(F)$ also depends only on G_F; in [**1**, III.2.7] the question was raised whether or not h depends only on G_F. The aim of this article is to answer this question positively:

THEOREM. *The homomorphism h depends only on G_F, and not on the particular field F.*

Here is a sketch of the proof. One reduces to proving that, for any finite separable extension E of F, the 'Scharlau transfer' $T : W(E) \to W(F)$ given by $T(q)(x) = \mathrm{Tr}_{E/F} q(x)$ depends only on G_F and G_E. To

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