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ORTHOGONAL DECOMPOSITIONS OF INDEFINITE QUADRATIC FORMS

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Introduction. A well known theorem of Milnor (see [**8**] or [**9**]) classifies the unimodular indefinite quadratic forms over **Z**. Either the form represents both even and odd numbers, in which case the form diagonalizes as $\langle \pm 1, \dots, \pm 1\rangle$; or the form only represents even numbers, in which case it decomposes into an orthogonal sum of hyperbolic planes and 8-dimensional unimodular definite forms. We give here some generalizations of this theorem for indefinite forms with square free discriminant of rank at least three.

Let L be a **Z**-lattice on an indefinite regular quadratic **Q**-space V of finite dimension $n \geq 3$ with associated symmetric bilinear form $f : V \times V \to \mathbf{Q}$. Assume, for convenience, that $f(L, L) = \mathbf{Z}$ and that the signature $s = s(L)$ of the form is non-negative. Let $x_1, \dots, x_n$ be a **Z**-basis for L and put $d = dL = \det f(x_i, x_j)$, the discriminant of the lattice L. We assume that d is square free. Let $\langle a_1, \dots, a_n\rangle$ denote the **Z**-lattice $\mathbf{Z}x_1 \perp \cdots \perp \mathbf{Z}x_n$ with an orthogonal basis where $f(x_i) = f(x_i, x_i) = a_i$, $1 \leq i \leq n$. Most of our notation follows O'Meara [**7**]. Thus L_p denotes the localization of L at the prime p.

The lattice L is called *even* if $f(x) \in 2\mathbf{Z}$ for all $x \in L$; otherwise the lattice is *odd.* The condition that L is an odd lattice is equivalent to the local condition that L_2 diagonalizes over the 2-adic integers (since d is not divisible by 4).

Odd lattices. While not all odd indefinite lattices have an orthogonal basis, we can get very close to this.

THEOREM 1. *Let L be an odd indefinite* **Z***-lattice of rank $n \geq 3$ with square free discriminant d. Then*

$$L = \langle \pm 1, \dots, \pm 1\rangle \perp B,$$

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