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A GOOD λ INEQUALITY FOR DOUBLE LAYER POTENTIALS OF SURFACES THAT ARE NOT LIPSCHITZ

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Introduction. In this paper we prove a good-λ inequality for the double layer potential operators. These have the form

$$C_\varepsilon^j(A,f)(x) = \int_{|x-y|>\varepsilon} \frac{(x_j - y_j)f(y)}{(|x-y|^2 + (A(x)-A(y))^2)^{(n+1)/2}} dy,$$

where x, y are in $\mathbf{R}^n$. The corresponding Maximal Operator is

$$C_*^j(A,f)(x) = \sup_{\varepsilon>0} |c_\varepsilon^j(A,f)(x)|$$

The hypersurface $t = A(x)$ is not assumed to be Lipschitz. The Good-λ inequality that we will prove can be used to obtain weighted L^p estimates for the Double Layer Potential Operators as was done in the one dimensional case for the Cauchy Integral Operator in [**4**].

Statement and proof of the main result. Throughout this paper we will consider the real number p fixed and strictly larger than n. With such a p let

$$A^*(x) = ((|\text{grad}\,(A)|^p)^*(x))^{1/p} = M_p(|\text{grad}\,(A)|)(x),$$

where $(\ )^*$ denotes the maximal function and M_p is the p-maximal function. We are assuming that $|\text{grad}\,(A)|^p$ is locally integrable and that $A^*(x)$ is finite a.e.

With this notation we will prove

THEOREM 1. *There exists a constant k such that, for all positive ε, one can find a constant C_ε such that the following holds:*

$$|\{x : C_*^j(A,f)(x) > (1+\varepsilon)\lambda \;\&\; (1+A^*(x))^k f^*(x) \le \lambda/C_\varepsilon\}| < 0.9|\{x : C_*^j(A,f)(x)\rangle\lambda\}|,$$

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