ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 19, Number 2, Spring 1989

INVERSE LIMIT MEANS ARE NOT PRESERVED UNDER HOMEOMORPHISMS

B.E. WILDER

1. Introduction. A *mean* on a topological space, M, is a continuous mapping, μ, from $M \times M$ to M such that $\mu(x,y) = \mu(y,x)$ and $\mu(x,x) = x$ for all $x, y \in M$. If $M = \lim\{I_n, f_n\}$ is an inverse limit space, each coordinate space I_n admits a mean μ_n, and, for each n, the functional equation $f_n[\mu_{n+1}(x,y)] = \mu_n[f_n(x), f_n(y)]$ holds for all $x, y \in I_{n+1}$, then the sequence $\{\mu_n\}$ generates a mean, μ, on the space M, where $\mu(\{x_n\},\{y_n\}) = \{\mu_n(x_n,y_n)\}$. A mean generated in this manner is referred to as an *inverse limit mean* with respect to sequence $\{I_n, f_n\}$. Professor John Baker and the author **[1]** investigated this notion, and constructed an inverse limit continuum, M, in which each coordinate space, I_n, was the unit interval $[0,1]$ such that M did not admit an inverse limit mean. We were unable to show, however, that M did not admit a mean. This led us to formulate the following question: If $M = \lim\{I_n, f_n\}$, where each $I_n = [0,1]$ and M admits a mean, μ, is μ necessarily an inverse limit mean with respect to its sequence $\{I_n, f_n\}$? A related question would be if two such inverse limit spces are homeomorphic, and one admits an inverse limit mean with respect to its sequence, must the other space also admit on with respect to its sequence? The purpose of this paper is to answer both of these questions in the negative. For brevity, the definitions, terms and notations of **[1]** will be employed as in that paper and will not be repeated here. In what follows,g will denote the continuous function from $I = [0,1]$ onto I where

$$g(x) = \begin{cases} 2x & \text{if } 0 \le x \le 1/2, \\ 2-2x & \text{if } 1/2 < x \le 1. \end{cases}$$

2. A preliminary theorem. Theorem 4 of **[1]** established that there does not exist an *infinite* sequence $\{\mu_n\}$ of means on I such that,

1980 *Mathematics Subject Classification*: Primary 39B10, 39B30.
Received by the editors on January 29, 1986 and in revised form on February 27, 1986.