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ON WEAKLY ANALYTIC SUBSETS OF ℓ^∞

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Let $\Delta_0 = \{-1,1\}^{\mathbf{N}}$, considered as a subset of ℓ^∞. In [4], M. Talagrand shows that there exists a non weakly Borel subset of Δ_0. This result answered the following question of G. Edgar [2]: Does there exist a Banach space X such that the norm Borel sets are not equal to the weak Borel sets?

The purpose of this paper is to show that, although there exist non weakly Borel subsets of Δ_0, every subset of Δ_0 is weakly analytic. That is, every subset of Δ_0 is the continuous image of a weakly Borel subset of Δ_0.

Our notation and terminology follow [1]. By the Cantor set, we mean the set $\Delta = \{-1,1\}^{\mathbf{N}}$ with the topology of coordinate convergence. For each n there is a natural partition π_n of Δ into 2^n open subsets: the elements of π_n are obtained by fixing the first n coordinates. This gives rise to 2^{2^n} different continuous functions $\varphi : \Delta \to \{-1,1\}$ which are constant on the members of π_n. The totality of all such functions φ (over all $n = 1,2,3,\ldots$) forms a countable set. Let $(\varphi_n)_{n=1}^\infty$ be an enumeration of these functions.

A subsequence of $(\varphi_n)_{n=1}^\infty$ is the sequence $(r_n)_{n=1}^\infty$ of Rademacher functions, defined by

$$r_n(\varepsilon) = \varepsilon_n \quad \text{for } \varepsilon \in \Delta.$$

We shall assume for later that the enumeration above was chosen so that

$$\varphi_{2n} = r_{2n}, \quad \text{for every } n.$$

The following two observations are easily proved.

(1) If $\mu \in C(\Delta)^*$, then the variation norm of μ is given by

$$||\mu|| = \sup_n |\langle \mu, \varphi_n \rangle|.$$

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