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ON UNIVARIATE CARDINAL INTERPOLATION BY SHIFTED SPLINES

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1. Introduction. The object of this paper is to study cardinal interpolation of bounded data by integer translates of shifted B-splines. To set notation, M_n will denote the centered univariate B-spline of order n and, for any function $g(x)$ of the real variable x and a fixed real number $\alpha, g_\alpha(x)$ will stand for $g(x+\alpha)$; $\hat{g}$ will denote the Fourier transform of g. $I_{n,\alpha}f$ will represent the interpolant $\sum_{j\in \mathbf{Z}} a_j M_{n,\alpha}(\cdot - j)$ which agrees with a given function f on $\mathbf{Z}$ and $P_{n,\alpha}(x)$ will stand for the *characteristic polynomial*, viz.,

$$P_{n,\alpha}(x) = \sum_{j\in Z} M_{n,\alpha}(j)e^{-ijx}. \tag{1.1}$$

$I_{n,\alpha}f$ can also be written in the *Lagrange form*

$$I_{n,\alpha}f = \sum_{j\in Z} f(j)L_{n,\alpha}(\cdot - j), \tag{1.2}$$

where $L_{n,\alpha}$ is the *fundamental function* of interpolation.

An application of the Poisson summation formula to (1.1) yields the useful identity

$$\begin{aligned} P_{n,\alpha}(x) &= \sum_{j\in Z} \hat{M}_{n,\alpha}(x+2\pi j) \\ &= \sum_{j\in Z} \hat{M}_n(x+2\pi j)e^{i\alpha(x+2\pi j)}. \end{aligned} \tag{1.3}$$

It should also be recalled that the Fourier transforms of $L_{n,\alpha}$ and M_n are given by

$$\hat{L}_{n,\alpha}(x) = \frac{\hat{M}_{n,\alpha}(x)}{P_{n,\alpha}(x)} \tag{1.4}$$

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