LINEAR ALGEBRA IN THE CATEGORY
OF C(M)-LOCALLY CONVEX MODULES

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ABSTRACT. This paper is concerned with the "linear algebra" of Banach bundles (otherwise known as bundles of Banach spaces). Although it is not entirely self-contained, it furnishes a unified, detailed treatment of the category Bun_X consisting of all Banach bundles $\pi : E \to X$ (X a compact Hausdorff space). For the most part, we accomplish our study through a formulation of results for the equivalent (and, for our purposes more convenient) category LCMod_X whose objects are the $C(X)-$locally convex $C(X)$-modules.

Many results are new (such as the study of projective and injective objects in the category): some topics, like tensor products of Banach bundles, have been studied previously, but are developed further here. In other cases, "known" facts (that Bun_X is a cocomplete category, for instance), which have appeared in the literature in scattered and sometimes sketchy form, receive a systematic exposition.

Introduction. Throughout this paper, the space X mentioned in the title will be both compact and Hausdorff. We will direct our attention toward two naturally equivalent categories of Banach structures over X: the category LCMod_X, whose objects are the $C(X)$-locally convex modules over $C(X)$, and the category Bun_X of all Banach bundles (i.e., bundles of Banach spaces) over X.

Our chief interest will be in the construction of new objects from old in these categories - limits, colimits, tensor products, hom sets, injective envelopes, etc. These constructions involve not only the objects of these categories, but also their morphisms. Not surprisingly, therefore, several important functors become involved.

This paper continues the study, begun by the authors in earlier papers (see Kitchen and Robbins [10], [11], and [12]), of the relationship