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ON THE RELATIVE GROWTH OF AREA FOR SUBORDINATE FUNCTIONS

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Introduction. Let f be analytic in the open unit disk Δ and let $A(r, f)$ denote the area of the region on the Riemann surface onto which the disk $|z| < r$ is mapped by f. Then

$$A(r, f) = \int_{|z|<r}\int |f(z)|^2 dxdy = \pi \sum_{n=1}^{\infty} n|a_n|^2 r^{2n}.$$

If F is also analytic in Δ, we say f is *subordinate* to $F(f \prec F)$ if there exists a bounded analytic function ω, $\omega(0) = 0$, such that $f(z) = F(\omega(z)), z \in \Delta$. Golusin [**5**] has shown that if $f \prec F$, then

$$A(r, f) \leq A(r, F), \quad r \leq 1/\sqrt{2}.$$

Reich [**6**] has extended this result by showing that, for $0 < r < 1$,

$$(1) \qquad A(r, f) \leq T(r)A(r, F),$$

where

$$T(r) = mr^{2m-2}$$

in the range

$$\frac{m-1}{m} \leq r^2 \leq \frac{m}{m+1} \quad (m = 1, 2, \dots).$$

He also finds, for each r, all pairs $(f; F)$ for which equality holds in (1). Waniurski and this author [**3**] have extended Reich's results to quasi-subordinate pairs. It is the purpose, however, of this paper to examine

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