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CONTINUOUS APPROXIMATION METHODS FOR THE REGULARIZATION AND SMOOTHING OF INTEGRAL TRANSFORMS

R.P. BENNELL AND J.C. MASON

ABSTRACT. Continuous approximation methods are described for obtaining a numerical solution $f(t)$ to an integral transform $g(s) = \int K(s,t)f(t)dt$, where the given function $g(s)$ may be affected by noise and where the problem may be ill-posed. The approximate solution is expressed in the linear form $f^* = \sum a_j\phi_j$, where a_j are parameters and ϕ_j are certain basis functions; the values of a_j are determined by the minimization of a regularising/smoothing measure, which takes account of both the discrete l_2 error in the integral transform and the continuous L_2 norm of f^* or of one of its derivatives. A Generalized Cross-Validation technique, based on the work of G. Wahba, is used for determining the smoothing parameter, and efficient algorithms are developed for three specific sets of basis functions $\{\phi_j\}$, including a novel algorithm when $\{\phi_j\}$ are chosen to be a set of eigenfunctions. Numerical examples are given to compare the merits of the various algorithms. In the case where the function $g(s)$ is not affected by noise, the established "Method of Truncated Solutions" is adopted and an improved version of this method, based on B-splines, is described and then tested on numerical examples.

1. Introduction. Consider the Fredholm integral equation of the first kind

$$\int_a^b K(s,t)f(t)dt = g(s), \qquad c \le s \le d, \tag{1}$$

where $K(s,t)$ is a given kernel and $g(s)$ is a given function, (the range $[c,d]$ of values of s, does not necessarily coincide with the range of integration $[a,b]$).

We may write (1) in the form of an operator equation

$$Kf = g \quad \text{where } K : F \to G, \tag{2}$$

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