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ASYMPTOTICS FOR ORTHOGONAL POLYNOMIALS WITH REGULARLY VARYING RECURRENCE COEFFICIENTS

WALTER VAN ASSCHE[1] AND JEFFREY S. GERONIMO[2]

1. Introduction. Let $\{p_n(x); n = 0, 1, 2, \ldots\}$ be orthogonal polynomials with recurrence relation

$$xp_n(x) = a_{n+1}p_{n+1}(x) + b_n p_n(x) + a_n p_{n-1}(x), \quad n = 0, 1, 2, \ldots, \quad p_{-1} = 0, \quad p_0 = 1. \tag{1.1}$$

We want to investigate the case where the recurrence coefficients a_n and b_n are unbounded. We will use the notion of regular variation to specify the behavior of a_n and b_n as n tends to infinity.

DEFINITION. (SENETA [**20**, p. 46]) A sequence $\{c_n : n = 0, 1, 2, \ldots\}$ is regularly varying at infinity if there exists a sequence $\{\lambda_n : n = 0, 1, 2, \ldots\}$ such that

$$\lim_{n\to\infty} \frac{c_n}{\lambda_n} = 1$$

and

$$\lim_{n\to\infty} n\left(\frac{\lambda_{n+1}}{\lambda_n} - 1\right) = \alpha \tag{1.2}$$

The number α is called the index of regular variation.

One can show that a regularly varying sequence $\{c_n : n = 0, 1, 2, \ldots\}$ with index α can be written as $c_n = n^\alpha L(n)$ where L is a positive and measurable function on $[0, \infty)$ such that, for every $y > 0$,

$$\lim_{x\to\infty} \frac{L(xy)}{L(x)} = 1$$

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