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EXTENSION OF A THEOREM OF LAGUERRE TO ENTIRE FUNCTIONS OF EXPONENTIAL TYPE II

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1. Introduction. A domain whose boundary is a circle or a straight line is called a circular domain. The following theorem of Laguerre [**6**, pp. 56-63] which we state in a form used in [**12**, p. 33] does not only play an important role in the location of critical points of polynomials [**7**] but also allows one to deduce Bernstein's inequality for polynomials on the unit disk and various of its refinements [**3**; **10**, Chapters 1 and 4].

THEOREM A. *Let $p(z)$ be a polynomial of degree $n \geq 1$. If $p(z) \neq 0$ in a (closed or open) circular domain K, then*

$$n\, p(z) - (z - \zeta)p'(z) \neq 0 \quad \text{for } z \in K,\ \zeta \in K$$

which in the case $\zeta = \infty$ means that $p'(z) \neq 0$ for $z \in K$.

With the object of getting a result of similar scope for entire functions of exponential type we recently [**11**] proved the following

THEOREM 1. *Let f be an entire function of exponential type $\tau > 0$ such that*

$$h_f(\pi/2) := \limsup_{r \to \infty} \frac{\log |f(re^{i\pi/2})|}{r} = 0$$

and denote by H the (closed or open) upper half-plane. If $f(z) \neq 0$ for $z \in H$, then

$$\tau f(z) + i(1 - \zeta)f'(z) \neq 0 \text{ for } z \in H \text{ and } |\zeta| \leq 1.$$

We showed that the assumptions of this theorem cannot be relaxed and it constitutes an extension of Theorem A. Furthermore, we deduced

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