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PROXIMINALITY OF CERTAIN SUBSPACES OF $C_b(S;E)$

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Throughout this paper, S is a completely regular Hausdorff space and E is a Banach space. The vector space of all continuous and bounded functions $f : S \to E$, denoted by $C_b(S;E)$, is equipped with the sup-norm

$$||f|| = \sup\{||f(x)||; x \in S\}.$$

Recall that a closed subpsace V of a Banach space E is said to be *proximinal* if every $a \in E$ admits a best approximant from V, i.e., a point $v \in V$ for which

$$||v - a|| = \inf\{||w - a||; w \in V\} = \text{dist}(a; V).$$

The set of best approximants to a from V is denoted by $P_V(a)$, and the set-valued mapping $a \to P_V(a)$ is called the *metric projection.* If V is proximinal, then $a \to P_V(a) \neq \emptyset$ for every $a \in E$. If $P_V(a)$ is a singleton for each $a \in E$, then V is called a *Chebyshev subspace* of E. If V is a proximinal subspace of E, then a map $s : E \to V$ such that $s(a)$ belongs to $P_V(a)$, for each $a \in E$, is called a *metric selection* or a *proximity map* for V.

The following notations are standard and will be used throughout this paper. If $a \in E$ and $r > 0$, $B(a;r) = \{v \in E; ||v - a|| < r\}$ and $\overline{B}(a;r) = \{v \in E : ||v - a|| \leq r\}$. For any $s \in S$, the bounded linear operator $\delta_s : C_b(S;E) \to E$ is defined by $\delta_s(f) = f(s)$, for all $f \in C_b(S;E)$. If W is a closed vector subspace of $C_b(S;E)$, then $\delta_s|W$ denotes the restriction of δ_s to W. Notice that $0 \leq ||\delta_s|W|| \leq 1$.

Given a proximinal subspace V of a Banach space E, then clearly $C_b(S;V)$ is a closed subspace of $C_b(S;E)$. In this paper we shall study the following questions.

QUESTION 1. Under what assumptions is $C_b(S;V)$ proximinal in $C_b(S;E)$?

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