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RATIONAL APPROXIMATION-ANALYSIS OF THE WORK OF PEKARSKIĬ

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ABSTRACT. We discuss two great papers by the Soviet mathematician A.A. Pekarskiĭ (Mat. Sb. **124** (1984), 571-588 and **127** (1985), 3-19), where a decisive step is made in the long open problem of describing the space of functions admitting a given rate of best *rational approximation*. We also indicate several directions for further work.

0. Introduction. The main objective of this talk is to report (§4 -§6) on two great papers by A.A. Pekarskiĭ [**16**], [**17**], which, as far as we can see, constitute a real breakthrough in the long open problem concerning the rate of best *rational approximation* in the L_p-metric.

Whereas much work has been devoted to the corresponding problem for *polynomial* approximation (originally only in the case of the uniform or Chebyshev norm; see any monograph on approximation theory), our understanding of *rational approximation* from this point of view has until recently been rather meager (we summarize some previous results in §2). The difficulty comes from the circumstance that the problem (in the case of *rational approximation*) is to some extent *non-linear* in nature (the set of rational functions of degree not exceeding a given number is not a vector space). From the abstract point of view of interpolation of normed Abelian groups [**15**], it is that in this case one has a *non-Archimedean* norm, not an *Archimedean* one.

Approximation theorists usually tend to work on an interval. Pekarskiĭ's result is formulated for the case of the unit circumference **T** and all functions are assumed to be analytic in the unit disk D (or rather distributional boundary values of functions analytic there; this latter assumption, however, is not so very restrictive as it sounds) so that function theoretic techniques becomes available. More precisely, his main result:

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