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n-CONVEXITY AND MAJORIZATION

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ABSTRACT. The fact that the n^{th} order divided difference of an $(n+2)$-convex function is a symmetric, convex function of its arguments, and is therefore Schur convex, allows us to apply the theory of Majorization in order to derive inequalities for such functions. Several consequences of this result are presented. In a separate section the theory of majorization is used to compute bounds on the derivatives of polynomials.

1. n-Convexity and Schur convexity. The first two definitions are given in [**2**].

DEFINITION 1. Let $x, y \in \mathbf{R}^{n+1}$ be given. We say that y is *majorized* by $x (y \prec x)$ if and only if $\sum_{i=0}^n x_i = \sum_{i=0}^n y_i$ and

$$(1) \qquad \sum_{i=0}^{k} x_{[i]} \geq \sum_{i=0}^{k} y_{[i]}, \quad k = 0, \ldots, n-1,$$

where $x_{[0]} \geq \cdots \geq x_{[n]}$ denotes a decreasing rearrangement of $x_0, \ldots, x_n$.

Numerous example of majorization are given in [**2**].

DEFINITION 2. Let $x, y \in \mathbf{R}^{n+1}$ be given. A function $\varphi : \mathbf{R}^{n+1} \to \mathbf{R}$ is called *Schur convex* if and only if $x \prec y \Rightarrow \varphi(x) \leq \varphi(y)$.

The next definition can be found in [**4**].

DEFINITION 3. A function f is $(n+2)$-convex on (a, b) if and only if, for all $a < x_0 < \cdots < x_{n+2} < b$, the divided differences $[x_0, \ldots, x_{n+2}]f$

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