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# APPROXIMATION BY CHENEY-SHARMA-KANTOROVIČ POLYNOMIALS IN THE $L_p$-METRIC

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**1. Properties of CSB-polynomials.** Based on the identity

(1.1)
$$\sum_{k=0}^{n} p_{nk}(x;\beta) := (1+n\beta)^{-n}\sum_{k=0}^{n}\binom{n}{k}x(x+k\beta)^{k-1}[1-x+(n-k)\beta]^{n-k} = 1,$$

$x \in I := [0,1], \beta \in \mathbf{R}, n \in \mathbf{N}$, a partition of unity originating from a more general identity of Jensen [**6**], Cheney and Sharma [**1**] associated with a bounded function $f : I \to \mathbf{R}$ the polynomial

$$(P_{n,\beta}f)(x) := \sum_{k=0}^{n} p_{nk}(x;\beta) f\Big(\frac{k}{n}\Big) \tag{1.2}$$

of degree $n$, depending on a parameter $\beta$ and reducing to the $n$-th Bernstein polynomial for $\beta = 0$. We shall refer to it as the $n$-th Cheney-Sharma-Bernstein polynomial (briefly: CSB-polynomial). The CSB-operators $P_{n,\beta}$ defined by (1.2) are positive, linear, polynomial and preserve, due to (1.1), constant functions. In [**1**] it is proved that the sequence $(P_{n,\beta})_{n\in\mathbf{N}}$ gives a positive polynomial approximation method on the space $C(I), \|\cdot\|_\infty$ (i.e. $\lim_{n\to\infty}\|f - P_{n,\beta}f\|_\infty = 0$ for all $f \in C(I)$) if the parameters $\beta$ are chosen to be nonnegative and are coupled with $n$ (i.e. $\beta = \beta_n$) in such a way that

$$n\beta_n \to 0 \text{ for } n \to \infty. \tag{1.3}$$

Using estimates in [**1**] it can easily be shown that

$$(P_{n,\beta_n}t)(x) = x + o\Big(\frac{1}{n}\Big), \tag{1.4}$$

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