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SUFFICIENT CONDITIONS FOR ASYMPTOTICS ASSOCIATED WITH WEIGHTED EXTREMAL PROBLEMS ON R

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ABSTRACT. We derive a sufficient condition for asymptotics as $n \to \infty$, for

$$E_{np}(W) := \inf\{\|(x^n + P(x))W(x)\|_{L_p(\mathbf{R})} : \deg(P) < n\},$$

where $1 < p < \infty$, and $W(x)$ is a weight function supported on $\mathbf{R}$. This will be used in a forthcoming paper to show that if $W_\alpha(x) := \exp(-|x|^\alpha)$, $x \in \mathbf{R}, \alpha > 0$, then, for $1 < p < \infty$,

$$\lim_{n\to\infty} E_{np}(W_\alpha)/\{(\beta_\alpha n^{1/\alpha}/2)^{n+1/p}e^{-n/\alpha}\} = 2K_p,$$

where β_α and K_p are constants depending only on α and p respectively.

1. Introduction. Let $W(x)$ be a measurable function, non-negative in $\mathbf{R}$, with all power moments finite, positive on a set of positive measure, and let

$$p_n(W^2;x) = \gamma_n x^n + \cdots, \ \gamma_n > 0,$$

denote the n^{th} orthonormal polynomial for $W^2(x)$ so that, for $m, n = 0, 1, 2, \ldots$,

$$\int_{-\infty}^{\infty} p_m(W^2;x)p_n(W^2;x)W^2(x)dx = \delta_{mn}.$$

Recently, Freud's conjecture concerning the asymptotic behaviour of γ_{n-1}/γ_n as $n \to \infty$ for the weight $\exp(-|x|^\alpha)$ was proved in full

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