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JACKSON TYPE THEOREMS IN APPROXIMATION BY RECIPROCALS OF POLYNOMIALS

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ABSTRACT. It was previously shown by the authors that Jackson type theorems hold for the case of approximating a continuous real-valued function f on a real interval by the *reciprocals* of complex polynomials. In this paper we extend these results to the general case when f is complex-valued.

1. Statement of results. Let $C^*[-\pi,\pi]$ denote the set of 2π-periodic continuous complex-valued functions and let $C[-1,1]$ denote the set of continuous complex-valued functions on [-1,1]. For any $f \in C^*[-\pi,\pi]$ (resp. $f \in C[-1,1]$) we denote by $E^*_{0n}(f)$ (resp. by $E_{0n}(f)$) the error in best uniform approximation of f on $[-\pi,\pi]$ (resp. on [-1,1]) by *reciprocals* of trigonometric (resp. algebraic) polynomials of degree $\leq n$ with complex coefficients.

Our goal is to prove the following Jackson type theorems.

THEOREM 1. *There exists a constant M such that for any $f \in C^*[-\pi,\pi]$,*

$$E^*_{0n}(f) \leq M\omega(f;n^{-1}), \quad n = 1,2,3,\ldots,$$

where $\omega(f;\delta)$ denotes the modulus of continuity of f on $[-\pi,\pi]$.

THEOREM 2. *There exists a constant M such that, for any $f \in C[-1,1]$,*

$$E_{0n}(f) \leq M\omega(f;n^{-1}), \quad n = 1,2,3,\ldots,$$

where $\omega(f;\delta)$ denotes the modulus of continuity of f on [-1,1].

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