ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 19, Number 1, Winter 1989

MONOTONE POLYNOMIAL APPROXIMATION IN L^p

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ABSTRACT. Jackson type estimates on the rate of approximation of monotone functions in $L^p[-1,1]$ by means of monotone polynomials are obtained. The estimates involve an L^p-modulus of continuity or equivalently a Peetre functional that weighs differently the behavior of the function in the middle of the interval and near the end points.

1. Introduction. The first significant Jackson type estimates for monotone polynomial approximation of a monotone function were obtained by Lorentz and Zeller [**5**] for approximation in the sup norm. Shvedov [**6**] extended these results to the case of approximating a monotone $f \in L^p[-1,1]$, $1 \leq p < \infty$. He showed that the order of approximation in the L^p-norm of such a function by means of monotone polynomials of degree $\leq n$ can be estimated by $\omega(f, 1/n)_p$ where $\omega(f,\cdot)_p$ denotes the modulus of continuity of f in the L^p-norm. Shvedov [**7**] went on to improve the above estimates by replacing $\omega(f, 1/n)_p$ by $\omega_2(f, 1/n)_p$. Moreover he showed that we cannot expect an estimate involving higher order moduli of smoothness with constants independent of f and n. Ivanov [**2,3**] introduced certain τ-moduli of smoothness in order to prove an L^p-estimate analogous to the Timan pointwise estimates for nonconstrained polynomial approximation. Recently [**4**] we used the τ-modulus to obtain L^p-estimates for monotone polynomial approximation.

In this note we will give a somewhat different estimate involving a recently defined new modulus of continuity due to Ditzian and Totik [**1**]. This modulus of continuity is in our opinion much more convenient to work with than the τ-moduli. We are indebted to Z.Ditzian for discussing with us some of the results in that yet unpublished paper [**1**].

Recieved by the editor on September 3, 1986.
Keywords and phrases: degree of monotone approximation, Jackson type estimates, L^p-modulus of continuity, Peetre kernel.