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CONTRACTION OF THE SCHUR ALGORITHM FOR FUNCTIONS BOUNDED IN THE UNIT DISK

WILLIAM B. JONES AND W.J. THRON

ABSTRACT. If in the Schur algorithm, some of the parameters γ_n vanish, then successive elements in the sequences (1) $\{D_n F_n\}$ and (2) $\{C_n/E_n\}$ will be equal to each other so that one cannot get continued fractions with the elements of (1) and (2) as approximants. It is also difficult to determine the degree of correspondence of the sequences (1) and (2) to series P and Q at 0 and ∞, respectively. For the contraction, continued fraction expansions can be obtained and the degree of correspondence can be computed, using the contraction, for all elements of the sequences (1) and (2).

1. Introduction. In 1907 Carathéodory investigated functions holomorphic on the unit disk and mapping it into the right half plane Re $\omega > 0$. In two articles [**4**] in 1917/18 J. Schur studied the related family

$$U := \left\{ f : f(z) \text{ holomorphic and } |f(z)| \le 1 \text{ for } |z| < 1 \right\}. \tag{1.1}$$

(For further historical remarks and references see [**1**].)

Schur's investigation was based on the algorithm: given $f_n \in U$, determine f_{n+1} by

$$f_{n+1} := t_n^{-1}(z, f_n). \tag{1.2}$$

Here

$$t_n(z, w) := \frac{\gamma_n + zw}{1 + \overline{\gamma}_n zw}, \quad n \ge 0, \tag{1.3}$$

and

$$\gamma_n = f_n(0). \tag{1.4}$$

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