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BIVARIATE CARDINAL INTERPOLATION ON THE 3-DIRECTION MESH: l^p-DATA

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The analogue of the unvariate cardinal spline theory of Schoenberg has been successfully carried out for bivariate box splines on a three direction mesh [**1**,**2**,**3**,**4**]. However, there is one result that had eluded us: The convergence theory for bivariate cardinal spline operators from $l^p(\mathbf{Z}^2)$ to $L^p(\mathbf{R}^2)$. In [**5**] it was shown that the sequence of univariate cardinal spline interpolants, indexed by degree, has uniformly bounded norm when considered as a sequence of operators from $l^p(\mathbf{Z})$ to $L^p(\mathbf{R}), 1 < p < \infty$, and that these operators converge strongly in $L^p(\mathbf{R})$ to the classical Whittaker cardinal series. The analogous result for the bivariate case has been established only in the relatively trivial case $p = 2$ [**1**]. The aim of this paper is to complete this result, at least in the case of equal direction multiplicities.

The (centered) box spline M_n corresponding to the three directions $e_1 = (1,0), e_2 = (0,1), e_3 = e_1 + e_2 = (1,1)$ with equal multiplicities n may be defined by its Fourier transform,

$$\hat{M}_n(x) = \prod_{\nu=1}^{3} (\operatorname{sinc}(xe_\nu/2))^n$$

where $\operatorname{sinc}(t) := \sin t/t$. Thus, M_n is the n-fold convolution of the piecewise linear "hat-function" which indicates clearly the connection between box splines and univariate cardinal splines.

It was shown in [**3**] that the trigonometric polynomial

$$P_n(x) := \sum_{j \in Z^2} M_n(j) e^{-ijx} = \sum_{j \in Z^2} \hat{M}_n(x + 2\pi j)$$

is strictly positive and attains its minimum at $(2\pi/3, 2\pi/3) \bmod 2\pi \mathbf{Z}^2$. This implies that cardinal interpolation with the translates of the

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