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A PROBLEM OF RUBEL CONCERNING APPROXIMATION ON UNBOUNDED SETS BY ENTIRE FUNCTIONS

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Let F be a closed subset of the finite complex plane $\mathbf{C}$. We call two functions f and g, defined on F, *equivalent*, in which case we write $f \sim g$, provided f and g are bounded (equivalently unbounded) on the same sequences. We shall consider only continuous functions and so we may restrict our attention to sequences $\{z_n\}$ such that $z_n \to \infty$. Thus $f \sim g$ if and only if: for any sequence $\{z_n\}$ in $F, f(z_n) \to \infty$ if and only if $g(z_n) \to \infty$.

By $H(F)$ we denote the set of functions holomorphic on (a neighborhood of) F, and we set $A(F) = C(F) \cap H(F^0)$. The closed set F is called an *Arakelyan set* if, for each $f \in A(F)$ and each constant $\varepsilon > 0$, there exists an entire function g such that $|f - g| < \varepsilon$ on F. In this terminology the celebrated Arakelyan Theorem [**2**] states that F is an Arakelyan set if and only if $\overline{\mathbf{C}} \backslash F$ is both connected and locally connected. For further results related to Arakelyan's theorem see [**4**] and [**5**].

Let us call a closed set F a *Rubel set* if, for each $f \in A(F)$, there exists an entire function g such that $f \sim g$. Clearly an Arakelyan set is always a Rubel set.

The notion of Rubel set was introduced by L.A. Rubel who called them weak Arakelyan sets. At the 1976 Symposium on Potential Theory at Durham, Rubel posed the problem of characterizing Rubel sets. This problem also appears to be related to another problem posed by Anderson and Rubel [**1**].

Goldstein [**6**] has given a condition which, in case $F^0 = \emptyset$, is necessary in order for F to be a Rubel set. If $F^0 \neq \emptyset$ the condition is no longer necessary. Nor is it sufficient, even for $F^0 = \emptyset$.

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