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SUDDEN SYMMETRY IN SIMULTANEOUS APPROXIMATION

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ABSTRACT. In the theory of simultaneous approximation to a set of n formal power series and using n rational functions with a common denominator along the lines of simultaneous Padé approximation of type II use the set $f(z), f(wz), f(w^2 z), \cdots, f(w^{n-1}z)$ where w is a primitive n-th root of unity and f belongs to one of three specific classes of hypergeometric functions. In the case that the approximants are calculated with the aid of the same number of coefficients from each of the series, the invariance of the n-tuple of functions under rotation over $2\pi/n$ is transferred to the denominator polynomial, which therefore turns out to be a polynomial in z^n.

1. Introduction. Let n be an arbitrary natural number, $n \geq 2$, and consider an n-tuple of formal power series over $\mathbf{C}$ given by

$$f_j(z) = \sum_{m \geq 0} c_{j,m} z^m, \quad c_{j,0} \neq 0 \quad (j = 1, 2, \cdots, n).$$

For any $(n+1)$-tuple of non-negative integers $(r_0, r_1, \cdots, r_n), s = r_0 + r_1 + \cdots + r_n$, we pose the approximation problem of finding polynomials $P_0(z), P_1(z), \cdots, P_n(z)$ over $\mathbf{C}$ satisfying

$$(1) \qquad \begin{cases} \deg P_j(z) \leq s - r_j & (j = 0, 1, \cdots, n) \\ P_0(z) f_j(z) - P_j(z) = O(z^{s+1}) \text{ as } z \to 0 & (j = 1, 2, \cdots, n) \end{cases}$$

It is well known that there exist several classes of functions such that this approximation problem – the polynomials usually are called the *type* II or *German* polynomials for the functions $1, f_1, \cdots, f_n$ - has a unique solution $(P_1(z)/P_0(z), P_2(z)/P_0(z), \cdots, P_n(z)/P_0(z))$ if only the condition $P_0(0) = 1$ is added; cf. **[3]**, **[5]**, **[6]**, **[8]** and for $n = 1$ cf. **[9]**.

As the inverted denominators of the approximants, i.e., $z^{s-r_0} P_0(1/z)$, can be identified as orthogonal polynomials in the setting of indefinite

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