ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 18, Number 4, Summer 1988

GROUPS OF ISOMETRIES ON OPERATOR ALGEBRAS II

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ABSTRACT. We show that, to each C_0-group ρ of isometries on a C^*-algebra A, there corresponds a C_O-group α of automorphisms on A, and a unitary cocycle u satisfying, $\rho(t)a = u(t)\alpha(t)a$, $t \in \mathbf{R}$, $a \in A$. It is shown, that the generator of ρ is of the form, $a \to i(Ha - aK)$, where H and K are (unbounded) self-adjoint operators.

Introduction. We study the polar decomposition of a C_O-group ρ of isometries on a C^*-algebra. It is used to obtain information about the infinitesimal generator of ρ, and the implementability of ρ. The case, where the algebra contains a unit, is considered in [**9**].

It is known [**5**], [**7**], that a linear isometry, mapping a C^*-algebra onto itself, can be decomposed into a Jordan-automorphism, followed by multiplication by a unitary. The unitary may be chosen in the multiplier algebra of A. This decomposition is called the *polar decomposition.*

We prove in §1 that, if ρ is a C_0-group of isometries on a factor $\mathcal{M}$, and $\rho(t)a = u(t)\alpha(t)a$, a in $\mathcal{M}$, is the polar decomposition of each $\rho(t)$, then α is a C_O^*-group of automorphisms on $\mathcal{M}$, and u is a σ-weakly continuous unitary α-cocycle $(u(s+t) = u(s)\alpha(t)u(t))$ in$\mathcal{M}$. The corresponding result for a C_O-group of isometries on a C^*-algebra is proved in §2. In §3, we give necessary and sufficient conditions for u to be a representation of the additive group of real numbers. We prove, in §4, that it is possible to choose a representation of A such that $\rho(t)a = U(t)aV(t)$, for a pair of unitary C_O-groups U and V. We study the infinitesimal generator of a group of this form, see also [**9**, §4]. In the final section, we consider the case where A is a C^*-algebra of compact operators.

Notation. Let X be a Banach space. A group on X is a homomor-

Research supported by the Danish Natural Science Research Council
Received by the editors on March 12, 1986, and in revised form on September 15, 1986.