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ERGODIC SEQUENCES AND A SUBSPACE OF B(G)

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ABSTRACT. J. Blum and B. Eisenberg studied conditions on a sequence $\{\mu_n\}$ of probability measures on a locally compact abelian group G which ensured that, for any strongly continuous unitary representation π of G on a Hilbert space H and for any $\xi \in H$, $\{\int_G \pi(x)\xi d\mu_n(x)\}$ converges to a G-invariant member of H. In this paper their result is (essentially) generalized to non-abelian G. The generalization involves $\mathbf{B}_I(G)$, the closure of the linear span of the coefficients of the irreducible representations of G; thus $\mathbf{B}_I(G)$ contains $\mathbf{AP}(G)$ always, and equals $\mathbf{A}(G)$ if G is compact or abelian. The relationships of $\mathbf{B}_I(G)$ to $\mathbf{AP}(G)$ and to $\mathbf{C}_0(G)$ are investigated and $\mathbf{B}_I(G)$ is identified for some non-abelian groups, in particular, for the Heisenberg group, for which $\mathbf{B}_I(G)$ is not an algebra.

1. Introduction. Let G be a locally compact abelian group. By *representation* of G, we shall mean a strongly (equivalently, weakly) continuous unitary representation π of G on a Hilbert space H (as in [**7**; §13.1]) The fixed point set of π is

$$H_f = \{\xi \in H : \pi(x)\xi = \xi \text{ for all } x \in G\}.$$

A sequence $\{\mu_n\}$ of probability measures on G is called a *a strong operator ergodic* (s.o. ergodic) sequence or a generalized summing sequence if, for every representation π of G on a Hilbert space H and for every $\xi \in H, \{\pi(\mu_n)\xi\}$ converges in norm to a member of H_f. It is readily seen (via [**10**, §23], for example) that $\{\mu_n\}$ is s.o. ergodic if and only if, for every representation π of G on H, $\pi(\mu_n) \to P$ in the strong operator topology, where P is the orthogonal projection onto H_f.

Blum and Eisenberg [**1**] proved the following interesting.

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