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THE TOPOLOGICAL CLASSIFICATION OF CUBIC CURVES

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ABSTRACT. The real plane cubic curves given by $Ax^3 + Bx^2y + Cxy^2 + Dy^3 + Ex^2 + Fxy + Gy^2 + Hx + Iy + J = 0$ are classified up to equivalence under the homeomorphism group of the plane.

Introduction. In a previous paper [**18**] the author briefly sketched the history of classifications of cubic curves and described the two major approaches to the problem: group theoretic and non-group theoretic. The author then presented a group theoretic classification based on the affine group of the plane. In this paper a group theoretic classification based on the homeomorphism group of the plane is presented. The non-group theoretic approach was dominated by Newton. The fact that Newton's early classifications were based on criteria far from topological can easily be seen by inspecting the graphs on pages 72-84 of Vol. II of Newton's works [**9**].

An important and fundamental problem associated with the Kleinian (group theoretic) approach to classification is the computation of invariants. In the present context the relevant question is: How can the equivalence class of the curve be determined from the coefficients? This question is not answered here and is left to a future paper. The classification is presented by exhibiting a complete set of equivalence class representatives.

In every analytic geometry course, it is taught that, for quadratic curves $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, there are precisely three (nondegenerate) affine classes: parabola, ellipse, and hyperbola. Even though the homeomorphisms of the plane form a much larger class of transformations, it is nevertheless the case that the (nondegenerate) homeomorphism classes of quadratic curves are the same as the affine classes. For cubic curves these classes are much different. For example,

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