

NONSCATTERED ZERO-DIMENSION REMAINDERS

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ABSTRACT. If a completely regular Hausdorff space X is locally compact, then the maximal compactification ϕX of X having zero-dimensional remainder has the property that $\phi X - X$ is non-scattered if and only if every compact metric space is a remainder of X . In this paper characterizations of when $\phi X - X$ is non-scattered are presented for the case when X is not locally compact. The results are related to conditions on $R(X)$, the set of points in X which do not possess compact neighborhoods, and they apply in case X is rimcompact so that ϕX is the Freudenthal compactification of X .

1. Introduction. Substantial attention (for example, see [2, 3, 6, 7, 13], etc.) has been devoted to the question of existence and properties of the compactification ϕX of a non-locally compact, completely regular Hausdorff space X , where the remainder $\phi X - X$ is zero-dimensional and ϕX is maximum with respect to this property. In case X is locally compact, ϕX always exists and it follows from [5] that $\phi X - X$ is non-scattered if and only if all compact metric spaces are remainders of X . The results of [5] are extended by Ünlü in [12]. The purpose of this paper is to characterize when $\phi X - X$ is non-scattered in case X is not locally compact. If $R(X)$, the set of points in X which do not possess compact neighborhoods (in X), is compact, we show that $\phi X - X$ is non-scattered if and only if each compact metric space M is an open subset of the remainder of some compactification $\alpha_M X$, where $\alpha_M X \leq \phi X$ in the lattice of compactifications of X . If $R(X)$ is locally compact, then $\phi X - X$ is non-scattered whenever each compact metric space M is a subset of some $\alpha_M X - X$, where $\alpha_M X \leq \phi X$. In both cases $\phi X - X$ is non-scattered whenever each compact metric space M is a subset of some $\alpha_M X - X$, where $\alpha_M X \leq \phi X$. In both cases $\phi X - X$ is non-scattered if and only if $\phi X - X$ contains a compact, non-scattered subset. Also, conditions internal to X are provided which characterize these properties in each case.

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