ON WEIGHTED ORLICZ SEQUENCE SPACES
AND THEIR SUBSPACES

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ABSTRACT. Weighted Orlicz sequence spaces $\ell^\phi(a)$ containing an isomorphic copy of ℓ^∞ and c_0 are characterized by means of suitable conditions on the Orlicz function ϕ and the weight sequence (a_n) for $a_n \to 0$. This extends a result of B. Turett [14] for Orlicz spaces $L^\phi(\mu)$ over atomless measures of the case of purely atomic probability measures. As an application, the spaces $\ell^\phi(a)$ which are B-convex are determined. Also, a question of W. Luxemburg [11] on inclusions of spaces $\ell^\phi(a)$ for sequences (a_n) slowly decreasing to 0 is answered.

1. **Introduction.** Recent years have seen a quite profound analysis of the relationship between Orlicz spaces and the spaces $\ell^p, 1 \leq p \leq \infty$. In this direction a well-known result is that every Orlicz space always contains an isomorphic copy of some ℓ^p. A deeper analysis by J. Lindenstrauss and L. Tzafriri [9, 10] determined the set of all numbers p such that ℓ^p can be isomorphically embedded into an Orlicz sequence space ℓ^ϕ. For Orlicz spaces of functions $L^\phi(\Omega)$, B. Turett [14, 15] has characterized, in terms of the Orlicz function ϕ, the spaces $L^\phi(\Omega)$ for atomless finite measures containing an isomorphic copy of ℓ^∞ and c_0.

In this paper, we analyze these topics for *weighted Orlicz sequence spaces* $\ell^\phi(a)$ when $a_n \to 0$ or $a_n \to \infty$. The results answer the following general question: for which class of weight sequences (a_n) can the suitable characterizations of the non-atomic case be extended to the spaces $\ell^\phi(a)$?

In a more precise way, we study the class of weighted Orlicz sequence spaces $\ell^\phi(a)$ where (a_n) is of finite sum and

$$\varliminf_{n \to \infty} \frac{\sum_{k=n+1}^{\infty} a_k}{a_n} > 0$$