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WEIGHTED INEQUALITIES FOR A VECTOR-VALUED STRONG MAXIMAL FUNCTION

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ABSTRACT. We show weighted weak type and strong type norm inequalities for a vector analogue of the strong maximal function.

1. Let f be a locally integrable function on $\mathbf{R}^n$, the strong maximal function $M_s f$ is defined by

$$M_s f(x) = \sup_{x \in R} \frac{1}{|R|} \int_R |f(y)| dy,$$

where the supremum is taken over all rectangles R in $\mathbf{R}^n$, with edges parallel to the coordinate axes. We shall denote this class of rectangles by $\mathcal{R}$.

If $1 < q < \infty$ and $f = (f_1, \dots, f_k, \dots)$ is a sequence of functions defined on $\mathbf{R}^n$, we say that f is ℓ^q-valued if $f(x) \in \ell^q$, that is

$$|f(x)|_q = \left\{ \sum_{k=1}^{\infty} |f_k(x)|^q \right\}^{1/q} < \infty.$$

For such f we define $M_s f = (M_s f_1, \dots, M_s f_k, \dots)$.

A weight function w will be a non-negative, locally integrable function on $\mathbf{R}^n$ and for measurable $E \subset \mathbf{R}^n$ we write $w(E) = \int_E w(x) dx$. We say $w \in A_p(\mathcal{R})$, $1 \le p < \infty$, if there is a constant C such that

$$\left(\frac{1}{|R|} \int_R w(x) dx\right) \left(\frac{1}{|R|} \int_R w(x)^{-1/(p-1)} dx\right)^{p-1} \le C$$

for all $R \in \mathcal{R}$. For $p = 1$ the second factor on the left is understood to be ess $\sup_{x \in R} w(x)^{-1}$.

In this note we shall prove the following: