ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 18, Number 3, Summer 1988

# BOUNDARY VALUE PROBLEMS FOR SEMILINEAR ELLIPTIC EQUATIONS OF ARBITRARY ORDER IN UNBOUNDED DOMAINS

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ABSTRACT. We study boundary value problems for equations of the form $Au = f(x,u)$, where $A$ is an elliptic operator of order $2m$. If $A$ has suitable properties, we can allow $f(x,u)$ to grow in $u$ to an arbitrarily high power. It is allowed to have exponential growth even when $2m < n$.

**1. Introduction.** We shall be concerned with boundary value problems of the form

$$A(x,D)u = f(x,u) \text{ in } \Omega, \tag{1.1}$$

$$B_j(x,D)u = 0 \text{ on } \partial\Omega,\ 1 \le j \le m, \tag{1.2}$$

where $A(x,D)$ is a uniformly elliptic operator of order $2m$ in a (bounded or unbounded) domain $\Omega \subset \mathbf{R}^n$, and the operators (1.2) cover it on $\partial\Omega$, the boundary of $\Omega$(cf. [10, p. 224]). If the coefficients of $A(x,D)$ and the $B_j(x,D)$ as well as $\partial\Omega$ are sufficiently regular, then for any $1 < p < \infty$ the estimate

$$||u||_{2m,p} \le C(||A(x,D)u||_p + ||u||_p) \tag{1.3}$$

holds for $u \in H^{2m,p}(\Omega)$ satisfying (1.2), where $||u||_{k,p}$ is the norm in the Sobolev space $H^{k,p}(\Omega)$ and $||u||_p$ is the $L^p(\Omega$ norm (cf. Agmon-Douglis-Nirenber [**1**]). We shall require more: that $A(x,D)$ is a bijective map of those $u \in H^{2m,p}(\Omega)$ satisfying (1.2) onto $L^p(\Omega)$. Sufficient conditions for this to hold can be found in [**2, 3, 6, 8, 15-17**]. We shall show that it is true for the Dirichlet problem for constant coefficient operators for which the corresponding polynomial does not vanish on $\mathbf{R}^n$ (cf. §2).

Concerning the function $f(x,u)$ we shall assume that

$$|f(x,u)| \le \sum_{k=1}^{\infty} V_k(x)|u|^{b_k}, \quad b_k \ge 0 \tag{1.4}$$

Received by the editors on October 28, 1985.