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BOUNDED AND ALMOST PERIODIC SOLUTIONS OF SEMI-LINEAR PARABOLIC EQUATIONS

JAMES R. WARD, JR.

0. Introduction. Let Ω be a bounded domain in $\mathbf{R}^N, N \geq 1$, of class $C^{2+\alpha}, 0 < \alpha < 1$, and let $F : \mathbf{R} \times \Omega \times \mathbf{R}^2 \to \mathbf{R}$ be a continuous function. The purpose of this paper is to discuss the existence of bounded and almost periodic (in time) solutions of nonlinear parabolic boundary problems with a possible time delay. In particular semi-linear problems of the form (0.1), (.02) or (0.1), (0.3) will be studied.

$$u_t - \Delta u = F(t, x, u(t, x), u(t-r, x)) \text{ in } \mathbf{R} \times \Omega \tag{0.1}$$

$$u(t, x) = 0 \text{ on } \mathbf{R} \times \partial\Omega \tag{0.2}$$

$$\frac{\partial u}{\partial \nu}(t, x) = 0 \text{ on } \mathbf{R} \times \partial\Omega. \tag{0.3}$$

Here $u = u(t, x)$ is a real valued function on $\mathbf{R} \times \overline{\Omega}, \Delta := \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2}$ is the N-dimensional Laplacian, $x = (x_1, \dots, x_N), r \geq 0$, and $\partial/\partial\nu$ indicates the outward normal derivative on $\partial\Omega$, the boundary of Ω.

Suppose $G : \mathbf{R} \times \mathbf{R}^n \to \mathbf{R}^n, n \geq 1, (t, x) \mapsto G(t, x)$, is continuous and uniformly almost periodic in t. It is a well known result of Amerio that if there is a compact set $K \subset \mathbf{R}^n$ such that for every $G^* \in \operatorname{Hull}(G)$ the ordinary differential equation

$$\dot{u} = G^*(t, u)$$

has a unique solution on $\mathbf{R}$ with range in K then each such solution is an almost periodic function; see [**4**] for a proof and further references. Amerio's result has been generalized in several ways for both ordinary differential equations and for abstract evolution equations. In the latter case an extension has been made by Dafermos [**3**] by using the concept of an almost periodic process, a two parameter family of maps related to the usual evolution operator; his results depend upon the existence of such a process on a complete metric space. Haraux [**6**] has also

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