ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 8, Number 2, Spring 1988

INSTABILITIES IN STEADY FLOWS OF TWO FLUIDS

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ABSTRACT. Steady shearing flows and convection, involving two immiscible liquids separated by an interface, will be discussed with particular emphasis on the case when the fluids have similar densities but different viscosities. Many interface positions are theoretically allowed but only a few are observed experimentally, thus motivating a study of their stability. Numerical computations and asymptotic analyses for the stability of various arrangements will be discussed.

1. Introduction. This paper is based on a lecture given at the Conference on Nonlinear PDE, and is a review of the author's involvement in the study of stability of flows of two immiscible fluids.

Examples of two-fluid flows arise in a variety of contexts. In the pipeline transport of very viscous oils (which, in the simplest case, may be modelled by Hagen-Poiseuille flow), it has been observed that the addition of a small amount of water greatly reduces the pressure drop required for transportation [**1**]. The resistance to the flow is expected to arise mainly from friction at the pipe wall, so that replacing the viscous fluid by a less viscous immiscible one just along the wall would lower the work required to transport the viscous oil. For horizontal pipelining, such an arrangement was thought to be possible if the densities of the fluids are similar. In fact, experiments showed that the water migrates to the pipe wall, thus shielding the oil from intense shearing [**2**]. Moreover, by using additives in the less viscous liquid, the pipe wall could be protected from corroding. A related example is the extrusion of two molten polymers vertically out of a pipe and cooling in air [**3, 4, 5, 6**]. Experimental data are variable for flows with equal volumes and for flows where there is a small amount of the less viscous fluid. Again, the less viscous fluid eventually encapsulates the more viscous fluid, when the fluids are otherwise similar (see Figure 1): this appeared to be independent of the initial arrangement. Phenomena of this type are of immediate application to the fiber industry, in the spinning of bicomponent fibers, which are important for their self-crimping characteristics. Another area of industrial importance is