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ON SPURIOUS NUMERICAL SOLUTIONS FOR NONLINEAR EIGENVALUE PROBLEMS

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1. Introduction. It is well known that discretization of the nonlinear eigenvalue problem

$$\begin{aligned} u''(x) + \lambda f(u) &= 0 \\ u(-1) = u(1) &= 0 \end{aligned} \tag{1}$$

leads to a system of equations

$$AU = \lambda F(U) \tag{2}$$

for which there may exist so-called "spurious solutions" in addition to the "numerically relevant solutions" (NRS) which approximate the solutions to (1). The branches of spurious or "numerically irrelevant solutions" (NIS) characteristically are located away from the origin in (λ, U)-space. As the mesh size of the discretization is decreased these branches will recede farther away from the origin while the branch of NRS will remain in approximately the same place. For practical computing purposes it if therefore not difficult to identify (or even avoid) spurious solutions. This, however, does not make the phenomenon any less interesting and it has been studied by various means. For example, in the recent work of Peitgen and Nussbaum sophisticated dynamical systems theory was brought to bear upon the problem of NIS for nonlinear elliptic eigenvalue problems as well as the closely related problem of special periodic solutions ($x(t+2) = x(-t) = -x(t)$) of the delay differential equation $\dot{x}(t) = -\alpha f(x(t-1))$ for odd f [**9**], [**10**], [**11**]. In what may be the eariliest reference to spurious solutions, Gaines [**6**] considered nonlinearities of the form $f(u, u')$. He showed that the discretized problem may have spurious solutions and proposed an algorithm which would find the numerically relevant solutions while avoiding the spurious ones. In another early analytical study Allgower [**1**] showed that spurious solutions may exist for as simple a nonlinearity as $f(u) = u^m, m > 1$. Later work includes

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