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ON SUPERLINEAR ELLIPTIC PROBLEMS WITH NONLINEARITIES INTERACTING ONLY WITH HIGHER EIGENVALUES

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1. Introduction. Consider the Dirichlet problem

$$-\Delta u = g(x,u) \text{ in } \Omega, \quad \mu = 0 \text{ on } \partial\Omega, \tag{1}$$

where Ω is a bounded smooth domain in $R^N, N \geq 2$. For the purposes of this section we shall assume that $g : \overline{\Omega} \times R \to R$ is continuous, where $\overline{\Omega}$ denotes the closure of Ω. Grosso modo problem (1) is said to be *superlinear at* $+\infty$ if

$$\lim_{s\to+\infty} \frac{g(x,s)}{s} = +\infty. \tag{2}$$

In [**1**] Ambrosetti and Rabinowitz proved the existence of a solution for problem (1) under a set of conditions which we discuss next. First, growth assumptions at $\pm\infty$ on g were made so as to guarantee that the Euler-Lagrange functional

$$\Phi(u) = \frac{1}{2}\int |\nabla u|^2 - \int G(x,u), \quad G(x,s) = \int_0^s g(x,\xi)d\xi \tag{3}$$

is well defined in $H_0^1(\Omega)$. (Integrals $\int$ are supposed to be taken over the whole of Ω, unless indicated otherwise.) The critical points of Φ are then the H_0^1 solutions of (1). Also, in [**1**], the following condition is assumed in order to ensure that Φ satisfies the Palais-Smale condition:

(4) There are numbers $\theta \in (0, 1/2)$ and $s_0 > 0$ such that
$$0 < G(x,s) \leq \theta s g(x,s) \quad \text{for } |s| \geq s_0, x \in \Omega.$$

We remark that condition (4) implies that the function $g(x,s)$ is superlinear in both directions, that is, at $\pm\infty$. Namely, (4) implies

$$\lim_{s\to\pm\infty} \frac{g(x,s)}{s} = +\infty.$$

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