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ON THE GEOMETRY OF LEVEL SETS OF POSITIVE SOLUTIONS OF SEMILINEAR ELLIPTIC EQUATIONS

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1. Introduction. In a recent paper [3] the authors showed how certain a priori lower bounds on positive solutions of the quasilinear boundary value problem

$$\begin{aligned} -\Delta &= f(u) \text{ in } \Omega \\ u &= 0 \text{ on } \Omega \end{aligned} \tag{1.1}$$

may be obtained which are similar to bounds for positive solutions of

$$\begin{aligned} -u'' &= f(u) \text{ in } (0,\pi) \\ u(0) &= u(\pi) = 0. \end{aligned} \tag{1.2}$$

The crucial assumption in obtaining these bounds for (1.1) was that the bounded domain $\Omega \subseteq \mathbf{R}^n$ satisfied certain symmetry conditions. In particular, if Ω satisfies the symmetry conditions of Gidas-Ni-Nirenberg [7] (see below for a precise formulation) with respect to the standard basis of $\mathbf{R}^n$, then the level sets $\Omega_c$of u,

$$\Omega_c = \{x \in \Omega : u(x) > c\},$$

are starlike (see [9]). This fact, together with an identity of Rellich (see [1]), then implied the desired a priori lower bound. The question then arose whether these level sets still would be starlike in case the symmetry conditions of Gidas-Ni-Nirenberg are satisfied with respect to a not necessarily orthogonal basis. This paper is addressed to this question and we show, using some group theoretic considerations that these level sets are indeed starlike. We hence obtain extensions of a result of Kawohl [9] on starlike level sets and of our a priori estimates in [3].

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