ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 18, Number 2, Spring 1988

AN INVARIANCE PRINCIPLE FOR A CLASS OF MONOTONE SYSTEMS AND APPLICATION TO DEGENERATE PARABOLIC EQUATIONS

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1. Introduction. By extending the concept of Liapunov functional to define a Liapunov operator in an ordered Banach space we develop a method of proving stabilization of solutions to a class of monotone systems. This method is employed to prove that solutions to certain degenerate parabolic equations approach equilibria as t approaches $+\infty$.

Given a function p and a semi dynamical system we define the *upper Liapunov operator*, $\overline{V}(p)$, to be the smallest (in the order of the Banach space) super solution greater than or equal to p. The *lower Liapunov operator*, $\underline{V}(p)$, is defined in an analogous way. These operators are used to squeeze points on trajectories onto equilibria when the semi dynamical system has certain monotonicity and stability properties. This idea is inspired in part by work of C. Dafermos [**5**]. The general parabolic equation which can be treated in this way has the form

$$(1.1)\qquad \begin{aligned} u_t &= [a(x,u,\phi(u)_x)+b(x,u)]_x \text{ on } [0,1]\times(0,\infty)\\ a(x,u,\phi(u)_x)&+b(x,u)=0 \text{ at } x=0,1, \text{ for all } t>0, \end{aligned}$$

where ϕ is increasing and $a(x,u,p)$ is increasing in p.

In this paper, for illustrative purposes, we restrict applications to equations of the form

$$(1.2)\qquad \begin{aligned} u_t &= ((u_x)^m + u^\alpha(u-1)V_x)_x \text{ on } [0,1]\times(0,\infty)\\ (u_x)^m &+ u^\alpha(u-1)V_x = 0 \text{ at } x=0,1, \text{ for all } t>0, \end{aligned}$$

where V is a specified potential (see Figure 1). More varied and general applications will appear elsewhere.

Equation (1.2) is related to a model for the movement of two species with very different rates of mobility (e.g., cows and grass) and these rates are governed by biological pressure.

Received by the editors on April 9, 1986.