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SINGLE-LAYER SOLUTIONS FOR THE DIRICHLET PROBLEM FOR A QUASILINEAR SINGULARLY PERTURBED SECOND ORDER SYSTEM

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ABSTRACT. A constructive existence proof is given for solutions of boundary layer type for the Dirichlet problem for the singularly perturbed quasilinear second order system of differential equations $\varepsilon d^2x/dt^2 = F(t,x,\varepsilon)dx/dt + g(t,x,\varepsilon)$ on a compact interval in the case that a boundary layer occurs at only one endpoint of that interval, subject to a generalized Coddington/Levinson condition and (in the general case of large boundary-layer jump) subject to the assumption that the matrix-valued function $F(t,x,0)$ is given in terms of a vector potential f as $F(t,x,0) = \nabla_x f(t,x)$. A proposed approximate solution, as provided by the O'Malley construction, is readily available throughout the entire compact interval. A direct construction is given for the Green function for the linearization of the problem about this proposed approximate solution. The resulting Green function representation for the linearization is used to prove the existence of an exact solution that is well-approximated by the given approximate solution, yielding precise and detailed information on the behavior of the resulting solution throughout the given compact interval. The construction of the Green function is patterned after that of Smith [**29**] for the scalar case and employs certain Riccati transformations so as to provide convenient representations for certain fundamental solutions *and for their inverses.* The quasilinear second order system studied here occurs in mathematical models for certain chemical reactors.

1. Introduction.

Consider the second-order system

$$\varepsilon\frac{d^2x}{dt^2} = F(t,x,\varepsilon)\frac{dx}{dt} + g(t,x,\varepsilon) \text{ for } 0 < t < 1, \tag{1.1}$$

for small positive values of $\varepsilon(\varepsilon \to 0+)$, subject to the Dirichlet boundary conditions

$$x(0,\varepsilon) = \alpha(\varepsilon) \text{ at } t = 0, \text{ and } x(1,\varepsilon) = \beta(\varepsilon) \text{ at } t = 1, \tag{1.2}$$

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