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ON ω-FILTERED VECTOR SPACES AND THEIR APPLICATION TO ABELIAN p-GROUPS: II

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1. Introduction. An *ω-filtered vector space* is ordinary vector space equipped with a descending chain of subspaces $\{X_n : n \in \omega\}$. A morphism between ω-filtered vector spaces X and Y is a linear map $f : X \to Y$ such that for all $n, f(X^n) \subseteq Y^n$. The key example of an ω-filtered vector space - over $\mathbf{Z}(p)$, the field of p elements - is, of course, the socle, $G[p] \overset{\text{def}}{=} \{x \in G : px = 0\}$, of an abelian p-group.

In [**10**] we began a systematic investigation of ω-filtered vector spaces over an arbitrary countable field. In this paper we continue that study, with emphasis this time on questions which can be answered with the help of additional set-theoretic axioms, but which cannot be settled on the basis of the usual, Zermelo-Franekel axioms of sets theory (denoted ZFC). We give applications of our results to the theory of abelian p-groups, particularly to Crawley's Problem.

§2 is concerned with the classification of ω_1-separable ω-filtered vector spaces of dimension $\aleph_1$. Our results parallel those which have been obtained for ω_1-separable abelian groups (cf. [**7, 8, 11**): there is a satisfactory structure theory when we assume Martin's Axiom (MA) plus the negation of the Continuum Hypothesis ($\neg$CH), and extreme pathology when we assume CH or the Axiom of Constructibility (V=L). Among the consequences of the structure theory (which holds under MA+$\neg$CH) are (1) every weakly ω_1-separable space of dimension $\aleph_1$ is ω_1-separable (Corollary 2.2) and C-decomposable (Corollary 2.3) and (2) every ω_1-separable ω-filtered vector space over $\mathbf{Z}(p)$ of dimension $\aleph_1$ is the socle of a $p^{\omega+1}$-projective p-group. None of these consequences are theorems of ZFC.

§3 deals with dense subspaces of small codimension in an ω-filtered vector space, and has application to Crawley's Problem on the unique ω-elongation of p-groups. The main theorems (3.3 and 3.8) construct large numbers of dense subspaces of codimension 1 in any non-

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