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GREAT SPHERE FIBRATIONS OF MANIFOLDS

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1. Introduction. Let E be a smooth, closed n–manifold which is

a) smoothly fibred by k–spheres, and

b) smoothly embedded in S^N (the unit N–sphere in R^{N+1}) so that these k–sphere fibres appear as *great* k–spheres in S^N. We say simply that E is *fibred by great k–spheres.* In this paper we study great sphere fibrations of certain manifolds E and examine what restrictions the geometric constraint (b) above places on the class of topological k–sphere fibrations of E.

A good example to keep in mind is that of 3–sphere fibrations of the 7 sphere. There are infinitely many topologically inequivalent smooth 3–sphere fibrations of the 7–sphere [**5**]. By Proposition 2.1 below, each such fibration may be pictured as a fibration by great 3–spheres, provided we choose a suitable embedding of the 7–sphere into a large dimensional sphere S^N. If we insist that the 7–sphere appear as the unit sphere in $\mathbf{R}^8$, then Gluck, Warner, and Yang have recently shown that every smooth fibration of it by great 3–spheres is topologically equivalent to the Hopf fibration [**2**].

This illustrates the general expectation, namely, when we lower the dimension of the sphere in which we permit the total space to be embedded or place geometric constraints on the total space we correspondingly restrict the bundles whose fibres can thus be made into great k–spheres. It is in this way that geometric theory departs from the topological theory.

If $E \subseteq S^N$ is fibred by great k–spheres there is a hierarchy of three questions, in increasing order of difficulty, which guides our study:

1) Given two such fibrations, are they topologically equivalent?

2) If they are topologically equivalent, is it possible to deform one to the other through a one–parameter family of such fibrations?

3) What is the homotopy type of the space of all such fibrations?

In general, even question 1 remains unanswered for all but the simplest cases; however, all three questions were completely answered for great circle fibrations of the round of 3–sphere [**1**].

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