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CLOSED FILTERS AND GRAPH–CLOSED MULTIFUNCTIONS IN CONVERGENCE SPACES

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ABSTRACT. We study closed filters which generalize notions of regular filters and closed sets. Applying closed filters, we refine some results of G. Choquet and R.E. Smithson on graph–closed multifunctions.

0. Introduction. It is well known that every graph–closed multifunction $\Gamma : Y \rightarrow X$ is closed–valued, has the closed–valued inverse Γ^{-1} and is compact–to–closed, i.e., $\Gamma(K)$ is closed whenever K is a compact subset of Y [**1**]. However, in general compact–to–closed (multi) functions (with closed–valued inverses) need not be graph–closed (see, e.g., [**7**; Example 3.5]). The equivalence may be obtained under additional assumptions, e.g., that Y is a Hausdorff locally compact space (Smithson [**10**]). Besides, it follows from [**8**] that if Y is Hausdorff and for every multifunction $\Gamma : Y \rightarrow X$ (with the closed–valued inverse Γ^{-1}) this equivalence holds, then Y is locally compact.

In what follows, it is shown (in greater generality) that graph–closedness can be expressed in terms of some corresponding properties of the images of filters. Namely, graph–closed multifunctions turn out to be exactly those multifunctions with closed–valued inverses that map compact filters into closed filters.

This characterization theorem is in line with some recent results (e.g., [**3**]) which show that the investigation of certain properties of multifunctions (e.g., subcontinuity, upper semi–continuity) can be reduced to the study of filters

1. Terminology and notation. Let X be a nonempty set. Denote by φX the collection of all filters on X and let $\overline{\varphi}X = \varphi X \bigcup \{2^X\}$. We say that filters $\mathcal{F}, \mathcal{G} \in \varphi X$ *meet* [**3**] if $F \bigcap G \neq \emptyset$ for every $F \in \mathcal{F}$ and $G \in \mathcal{G}$. If filters $\mathcal{F}$ and $\mathcal{G}$ meet, then the family $\{F \bigcap G : F \in \mathcal{F}, G \in \mathcal{G}\}$ is a base of the *supremum* filter $\mathcal{F} \vee \mathcal{G}$. Note that $\mathcal{F}$ and $\mathcal{G}$ meet if and

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